IOP-Fest, May 2026

The Proximity Prize: What it is and What is Currently Known

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Joint work with **Gal Arnon** and **Giacomo Fenzi**

eprint.iacr.org/2026/680

Papers We Will Discuss

First proximity gaps for general linear codes: RVW'13, AHIV'17, RZ'18, BKS'18, BGKS'20

BCIKS'20: **Proximity gaps** for Reed-Solomon codes

ZCF'23: **BaseFold**: Efficient Polynomial Commitment Schemes from Foldable Codes

ACFY'24: **WHIR**: Reed–Solomon Proximity Testing with Super-Fast Verification

Hab'24: **BaseFold in the list-decoding regime**

Hab'25: A note on mutual correlated agreement for Reed-Solomon codes

Zei'24: Khatam: Reducing the Communication Complexity of Code-Based SNARKs

GCXK'25: From List-Decodability to Proximity Gaps

BCHKS'25: **On proximity gaps for Reed–Solomon codes**

GG'25: **Optimal proximity gaps for subspace-design codes and random RS codes**

CS'25: On Reed–Solomon proximity gaps conjectures

DG'25: On the distribution of the distances of random words

FS'25: Small-field hash-based SNARGs are less sound than conjectured

KKH'26: Failure of proximity gaps close to capacity

proxember 2025

CGHLLPS'26: S-two whitepaper

ABF'26: **Open Problems in List Decoding and Correlated Agreement**

$1,000,000

initiative by
ethereum foundation

in prizes to prove (or disprove!) Reed-Solomon proximity gaps conjectures

An initiative by the Ethereum Foundation to advance the foundations of modern zkVMs.

(initiated by **Justin Drake**)

PRIZE JUDGES

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What is the proximity challenge?

A question about polynomials over finite fields

The challenge: prove that a certain conjecture is true

- Only elementary algebra (no Adeles, Schemes, Sheaves, …)

Why is the EF paying $1M for a proof ??

⇒ if true, better (provable) proof systems based on Reed-Solomon

EF's Goal: avoid conjectures, as much as possible

The grand challenges

The grand MCA challenge. We are given a Reed–Solomon code $\mathcal{C} := \mathsf{RS}[\mathbb{F}, \mathcal{L}, k]$ defined over some smooth evaluation domain $\mathcal{L} \subseteq \mathbb{F}$. The code has constant rate, and in particular the rate $\rho(\mathcal{C}) := k/|\mathcal{L}|$ is one of $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\}$.

$$\text{determine the largest } \delta_{\mathcal{C}}^* \in [0,1] \text{ such that } \varepsilon_{\mathsf{mca}}(\mathcal{C}, \delta_{\mathcal{C}}^*) \leq 1/2^{128},$$

assuming $|\mathbb{F}|$ is sufficiently large so that such a $\delta_{\mathcal{C}}^*$ exists.

The grand list decoding challenge. We are given a Reed–Solomon code $\mathcal{C}$ as in the grand MCA challenge. For a constant m,

$$\text{determine the largest } \delta_{\mathcal{C}}^* \in [0,1] \text{ such that } |\Lambda(\mathcal{C}^{\equiv m}, \delta_{\mathcal{C}}^*)| \leq |\mathbb{F}|/2^{128},$$

assuming $|\mathbb{F}|$ is sufficiently large so that such a $\delta_{\mathcal{C}}^*$ exists.

Why is this important? What does it even mean?

Notation

Let $\mathcal{C} = RS[\mathbb{F}, \mathcal{L}, k] \subseteq \mathbb{F}^{|\mathcal{L}|}$ be a **Reed-Solomon code**

with a **smooth eval. domain** $\mathcal{L}$ (i.e. $\mathcal{L}$ is a power-of-2 coset of a subgroup of $\mathbb{F}$)

and **rate** $\rho(\mathcal{C}) \coloneqq k/|\mathcal{L}| \in \{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\}$

$\delta_{\min}(\mathcal{C}) \coloneqq \frac{\Delta_{\min}(\mathcal{C})}{|\mathcal{L}|} \in [0,1]$: the **minimum relative distance** of $\mathcal{C}$

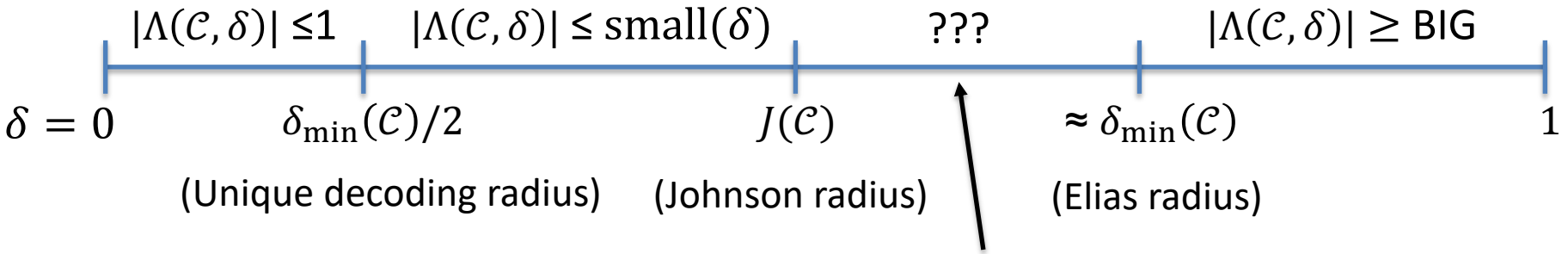
Recall: list decoding bounds for RS codes

The Johnson bound: For $\mathcal{C} \subseteq \mathbb{F}^n$ and $0 < \delta < J(\mathcal{C})$ $(= 1 - \sqrt{1 - \delta_{\min}(\mathcal{C})})$

$$\Lambda(\mathcal{C}, \delta) := \max_{w \in \mathbb{F}^n} |\{c \in \mathcal{C} : \Delta(w, c) \leq \delta\}| \leq \text{small}(\delta)$$

($\text{small}(\delta) \to \infty$ as δ approaches $J(\mathcal{C})$)

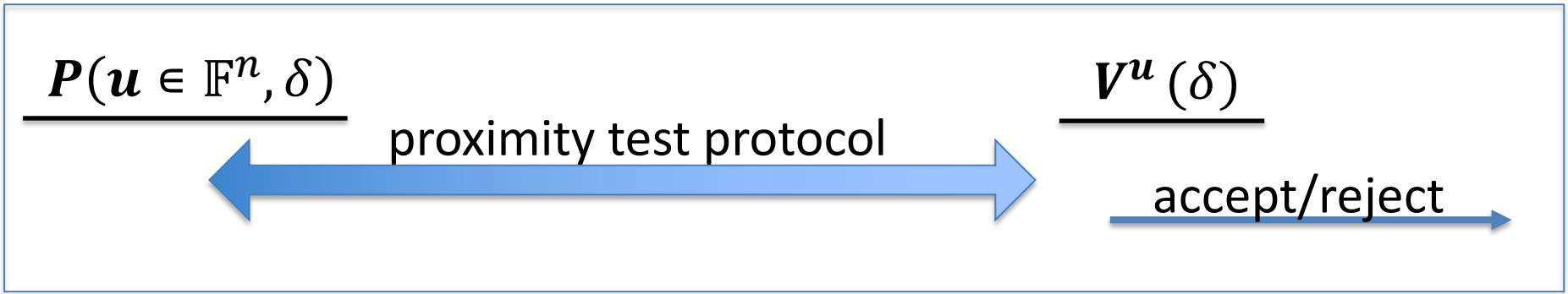
List decoding size $\Lambda(\mathcal{C}, \delta)$ as a function of $\delta \in [0,1]$:



Well behaved for Random-RS (RRS) and subspace-design codes, e.g. Folded-RS (FRS)

Proximity Testing: a SNARK ingredient

Let $\mathcal{C} \subseteq \mathbb{F}^n$ be a linear code, and $\delta \in [0, \delta_{\min}(\mathcal{C}))$.

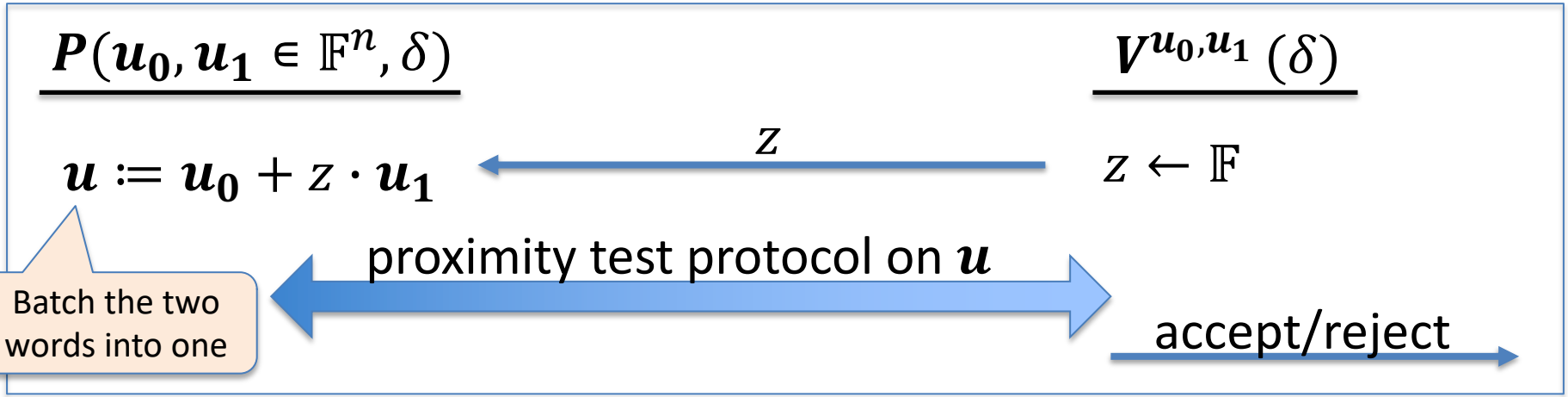


Completeness: if $u \in \mathcal{C}$ then Pr[Verifier accepts] = 1

Soundness: if u is δ-far from $\mathcal{C}$ then Pr[Verifier accepts] $< 1/2^{128}$

Batch proximity testing

Suppose we need two proximity tests. Can we batch them?



Complete: if $u_0, u_1 \in \mathcal{C}$ then $u \in \mathcal{C}$ so Pr[Verifier accepts] = 1

Is it sound? if one of u_0, u_1 is δ-far from $\mathcal{C}$, will V reject w.h.p ??

Answer: yes, if $\mathcal{C}$ has distance preservation.

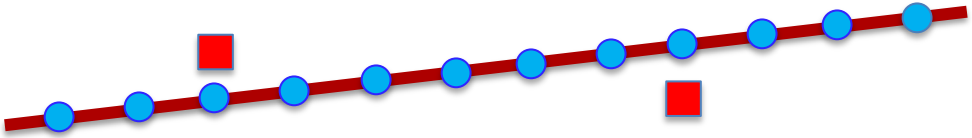
Distance preservation (proximity gap) [RVW'13]

Suppose one of $\boldsymbol{u_0}, \boldsymbol{u_1}$ is δ-far from $\mathcal{C}$.

Does a random linear combination preserve the distance?

Consider the line $L(z) = \boldsymbol{u_0} + z \cdot \boldsymbol{u_1}$

L:



we want at most few points on L to be δ-close to $\mathcal{C}$

Def: $p(L) := \Pr_{z \leftarrow \mathbb{F}}\left[\mathrm{dist}\left(L(z), \mathcal{C}\right) \leq \delta\right]$ (fraction of point on L that are δ-close to $\mathcal{C}$)

We say that $\mathcal{C}$ has **proximity gap** $\varepsilon_{\mathrm{pg}}(\mathcal{C}, \delta)$ if for every line L

$$p(L) \leq \varepsilon_{\mathrm{pg}}(\mathcal{C}, \delta) \quad \text{or} \quad p(L) = 1$$

Distance preservation (proximity gap)

For a given linear code $\mathcal{C}$,

what is the largest δ so that $\varepsilon_{\mathrm{pg}}(\mathcal{C}, \delta) < 1/2^{128}$??

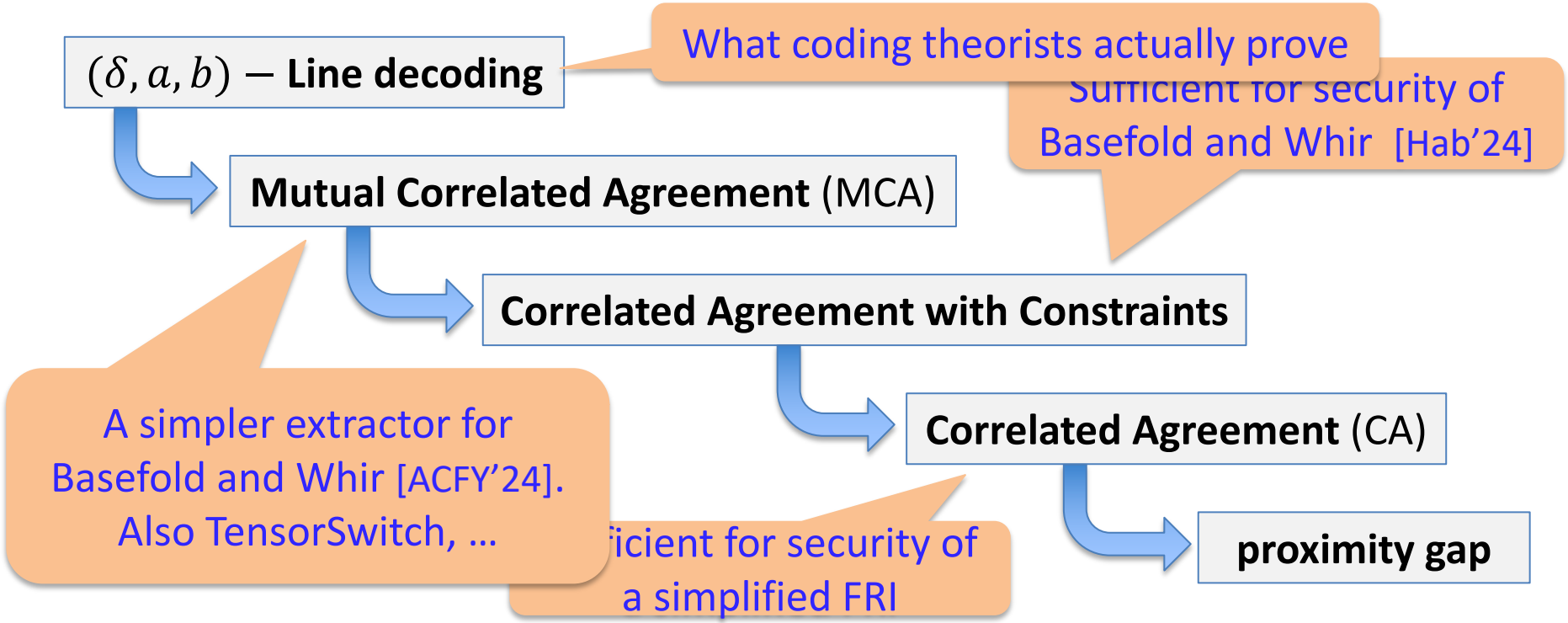
(a bigger δ results in a shorter SNARK proof --- as we will see)

A lot is known, but for for modern SNARKs

the basic proximity gap is insufficient … we need more.

Five proximity gap notions for a linear code $\mathcal{C}$

Proximity gap notions for $\mathcal{C}$ at some distance $\delta \in [0, \delta_{\min}(\mathcal{C}))$



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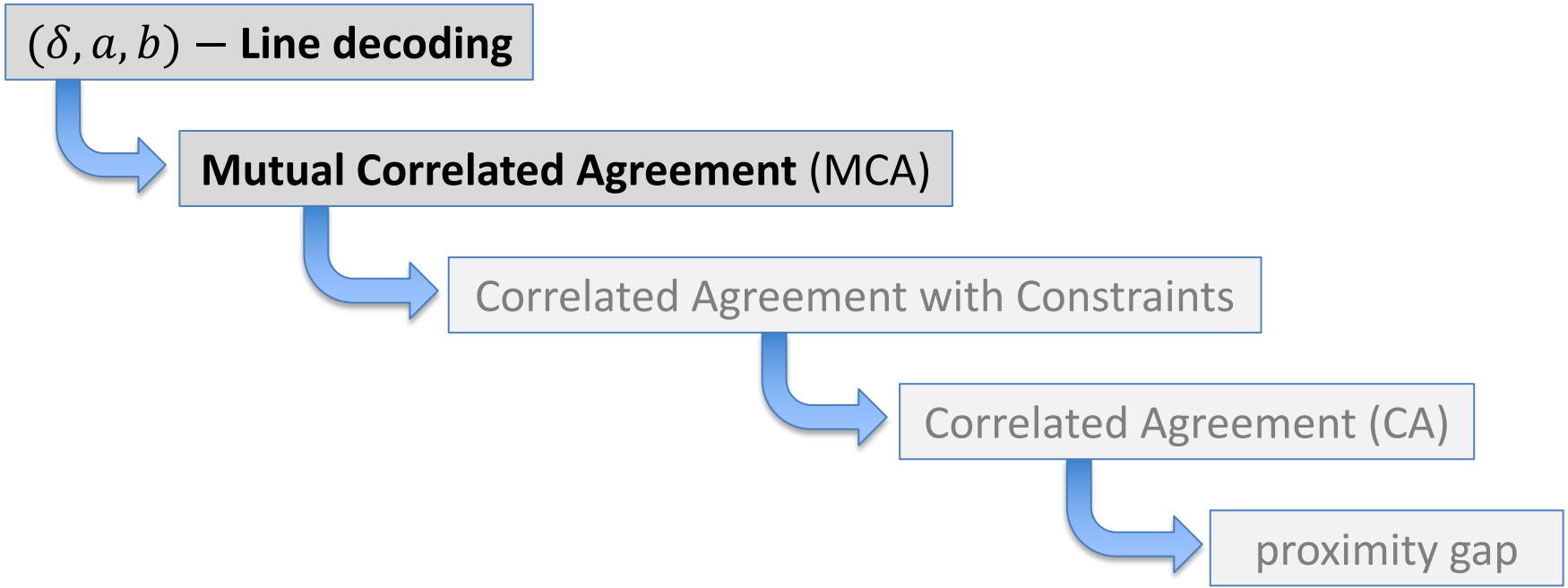
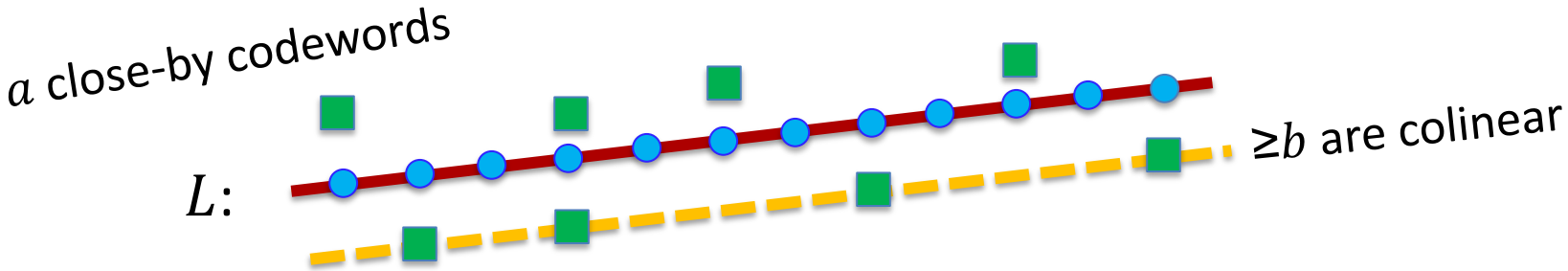


Our starting point: line decoding

Def: a linear code $\mathcal{C} \subseteq \mathbb{F}^n$ is (δ, a, b)-**line decodable** if for every line $L: \mathbb{F} \longrightarrow \mathbb{F}^n$ the following condition holds:

$\geq a$ points on L are δ-close to $\mathcal{C}$ $\Rightarrow$

≥b of the close-by codewords are colinear



(more generally, curve decodable)

GG'25: Goyal, Guruswami, eprint/2025/2054

CGH+'26: S-two paper, eprint/2026/532

Line decoding for Reed-Solomon Codes

Thm: [BCIKS'20, BCHKS'25, CGH+'26] Let $\mathcal{C} = RS[\mathbb{F}, \mathcal{L}, k] \subseteq \mathbb{F}^n$

Then $\mathcal{C}$ is (δ, a, b)-line decodable for $\delta \in \left(\frac{\delta_{min}(\mathcal{C})}{2}, J(\mathcal{C}) \right)$

unique decoding — Johnson bound

for $a \geq \eta(\delta)^5 \cdot n \ll |\mathbb{F}|$

and $b = \frac{a}{\eta(\delta)}$

where $\eta(\delta) \approx \max\left(\frac{1}{J(\mathcal{C})-\delta}, \text{ const}\right)$

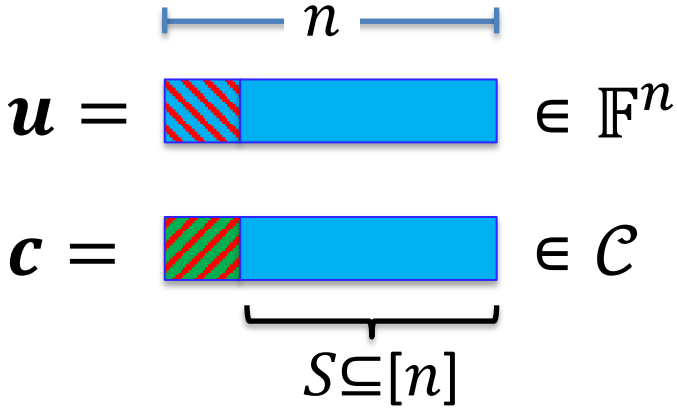
What about line decoding when $\delta > J(\mathcal{C})$? Good question!

CGH+'26: S-two paper, eprint/2026/532, Theorem 25

Mutual Correlated Agreement (MCA) [ACFY'25]

Let $\mathcal{C} \subseteq \mathbb{F}^n$ be a linear code, and $\delta \in [0, \delta_{\min}(\mathcal{C}))$.

Def: for $\boldsymbol{u} \in \mathbb{F}^n$ and $S \subseteq [n]$ we say that $\boldsymbol{u}$ **is** $(\boldsymbol{S}, \boldsymbol{\delta})$**-close to** $\boldsymbol{\mathcal{C}}$ if $|S| \geq (1-\delta)n$ and there is a $\boldsymbol{c} \in \mathcal{C}$ such that $\Delta_S(u, c) = 0$



$\boldsymbol{u}, \boldsymbol{v} \in \mathbb{F}^n$ are (S, δ)-close to $\mathcal{C}$

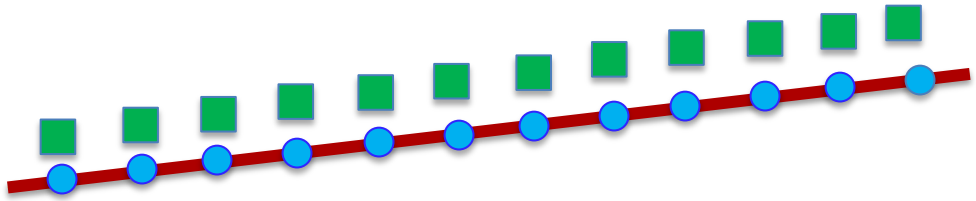
⇩

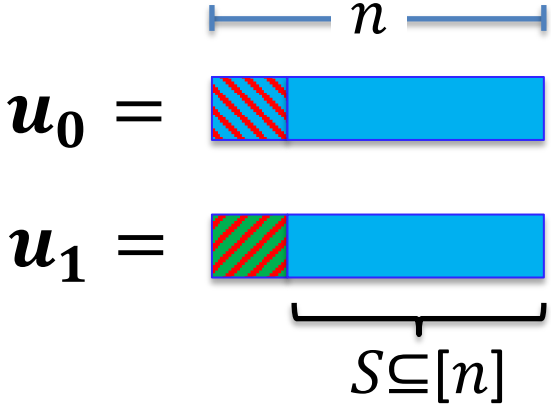
$\forall x, y \in \mathbb{F}$: $x \cdot \boldsymbol{u} + y \cdot \boldsymbol{v}$ is (S, δ)-close to $\mathcal{C}$

Mutual Correlated Agreement (MCA) [ACFY'25]

For $\boldsymbol{u_0}, \boldsymbol{u_1} \in \mathbb{F}^n$ consider the line $L(z) = \boldsymbol{u_0} + z \cdot \boldsymbol{u_1}$

L:





Clearly: if $\boldsymbol{u_0}, \boldsymbol{u_1}$ are (S, δ)-close to $\mathcal{C}$ then
$L(z)$ is (S, δ)-close to $\mathcal{C}$ for all $z \in \mathbb{F}$

MCA is the converse property:
If $L(z)$ is (S, δ)-close to $\mathcal{C}$ then w.h.p $\boldsymbol{u_0}, \boldsymbol{u_1}$ are (S, δ)-close to $\mathcal{C}$

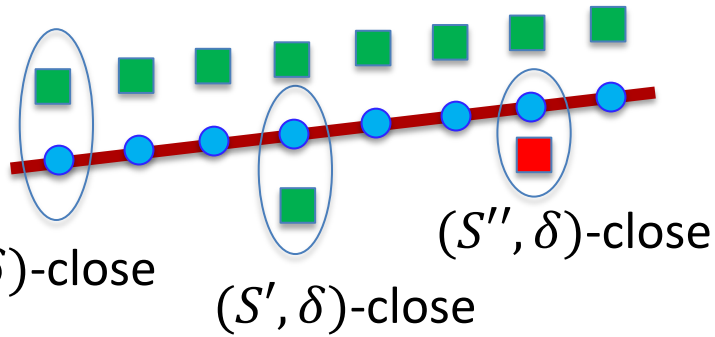
Mutual Correlated Agreement (MCA) [ACFY'25]

Def: $\mathcal{C} \subseteq \mathbb{F}^n$ has **Mutual Correlated Agreement** error $\varepsilon_{\text{mca}}(\mathcal{C}, \delta)$ if for every line $L(z) = \boldsymbol{u_0} + z \cdot \boldsymbol{u_1}$, $\boldsymbol{u_0}, \boldsymbol{u_1} \in \mathbb{F}^n$ we have:

for all $z \in \mathbb{F}$:

$L(z)$ is (S, δ)-close to $\mathcal{C}$ $\Rightarrow$ $\boldsymbol{u_0}, \boldsymbol{u_1}$ are (S, δ)-close to $\mathcal{C}$

except for $\leq \varepsilon_{\text{mca}}(\mathcal{C}, \delta)$ of the points on L (the MCA error)



$\Rightarrow$ $\boldsymbol{u_0}, \boldsymbol{u_1}$ are (S, δ)-close to $\mathcal{C}$

$\Rightarrow$ all $L(z)$ are (S, δ)-close to $\mathcal{C}$

$\Rightarrow$ $\boldsymbol{u_0}, \boldsymbol{u_1}$ are (S', δ)-close to $\mathcal{C}$

$\boldsymbol{u_0}, \boldsymbol{u_1}$ not (S'', δ)-close to $\mathcal{C}$ $\Rightarrow$ **MCA error**

From colinearity to agreement

Thm: [GG'25, Thm 3.5] for a linear code $\mathcal{C} \subseteq \mathbb{F}^n$

$$(\delta, a, n+1)\text{-line decodable} \quad \Rightarrow \quad \varepsilon_{\text{mca}}(\delta) < \frac{a}{|\mathbb{F}|}$$

Proof: Suppose some a points on the line $L(z) = u_0 + z\, u_1$ are MCA errors.

All these a points are δ-close to $\mathcal{C}$ $\Rightarrow$ $n+1$ of the close-by codewords are colinear

So: $\exists n+1$ points $\{\boldsymbol{p}_i \in L,\ (S_i, \delta)$-close to $\mathcal{C}\}$ and close-by codewords are colinear

For $n+1$ subsets of $[n]$ there must exist an i^* s.t. $S_{i^*} \subseteq \bigcup_{j \in [n+1] \setminus i^*} S_j$

By collinearity of close-by codewords: both $\boldsymbol{u_0}, \boldsymbol{u_1}$ are (S_{i^*}, δ)-close to $\mathcal{C}$.

… but then $\boldsymbol{p}_{i^*}$ is not an MCA error, contradiction! ■

The resulting MCA bound for RS codes

Let $\mathcal{C} = RS[\mathbb{F}, \mathcal{L}, k] \subseteq \mathbb{F}^{|\mathcal{L}|}$. For $\delta \in \left(\frac{\delta_{\min}(\mathcal{C})}{2}, J(\mathcal{C})\right)$

$$\varepsilon_{\text{mca}}(\mathcal{C}, \delta) < \frac{\eta(\delta)^5 \cdot |\mathcal{L}|}{|\mathbb{F}|} < 2^{-128}$$

for sufficiently large $|\mathbb{F}|$

What about ε_{mca} when $\delta > J(\mathcal{C})$???

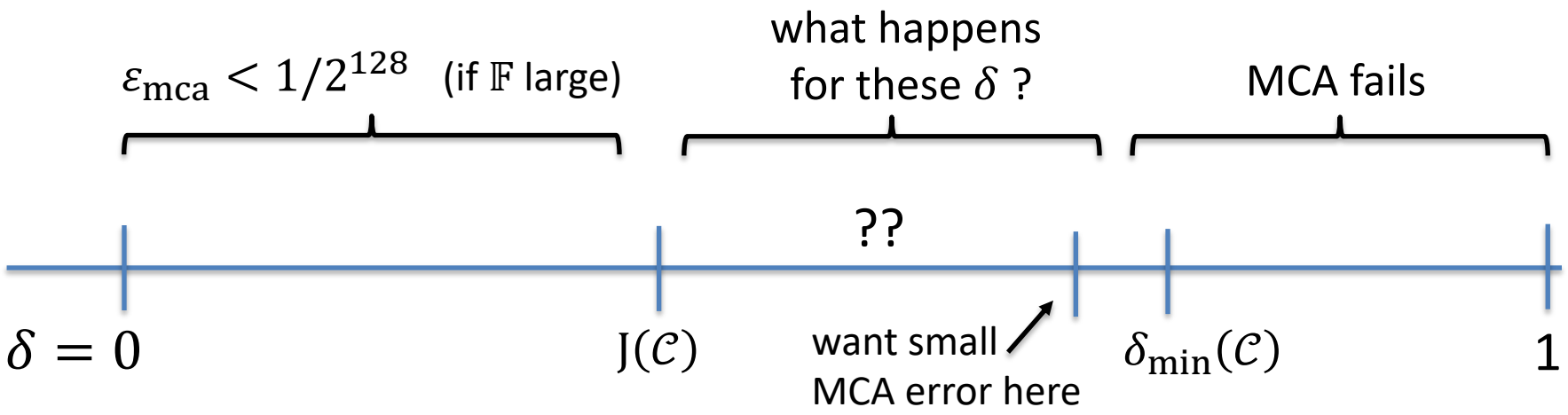
Habock'25: eprint/2025/2110

The Proximity Challenge for Constant Rate RS Codes

The grand MCA challenge. We are given a Reed–Solomon code $\mathcal{C} := \mathsf{RS}[\mathbb{F}, \mathcal{L}, k]$ defined over some smooth evaluation domain $\mathcal{L} \subseteq \mathbb{F}$. The code has constant rate, and in particular the rate $\rho(\mathcal{C}) := k/|\mathcal{L}|$ is one of $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\}$.

determine the largest $\delta_{\mathcal{C}}^* \in [0, 1]$ such that $\varepsilon_{\mathsf{mca}}(\mathcal{C}, \delta_{\mathcal{C}}^*) \leq 1/2^{128}$,

assuming $|\mathbb{F}|$ is sufficiently large so that such a $\delta_{\mathcal{C}}^*$ exists.



Summary for RS codes over a smooth domain

distance δ	error $\varepsilon_{\mathsf{mca}}(\mathcal{C}, \delta)$	reference
$\delta = 0$	$\varepsilon_{\mathsf{mca}}(\mathcal{C}, \delta) = \frac{1}{\lvert\mathbb{F}\rvert}$	trivial
$\delta < \delta_{\mathsf{min}}(\mathcal{C})/2$ (δ below the unique decoding radius)	$\varepsilon_{\mathsf{mca}}(\mathcal{C}, \delta) \leq \frac{O(n)}{\lvert\mathbb{F}\rvert}$	[ACFY25; BCIKS20]
$\delta = J(\delta_{\mathsf{min}}(\mathcal{C})) - \eta$ (as δ approaches the Johnson radius)	$\varepsilon_{\mathsf{mca}}(\mathcal{C}, \delta) \leq \frac{n \cdot \mathrm{poly}(1/\eta)}{\lvert\mathbb{F}\rvert}$	[BCHKS25; Hab25; BCGM25; BCIKS20]

The behavior of $\varepsilon_{\mathsf{mca}}(\mathcal{C}, \delta)$ for $\delta \in [0, 1]$ and $\mathcal{C} := \mathsf{RS}[\mathbb{F}, \mathcal{L}, k]$, where $n := \lvert\mathcal{L}\rvert$.

The KKH negative result: MCA fails when δ is too big (also CS'25, DG'25, FS'25)

Krachun, Kazanin, Haböck: eprint/2026/782

Line decoding for RRS and FRS

Thm: [GG'25] The following codes are line decodable **almost up to capacity** ($\delta_{\min}(\mathcal{C})$)

- Subspace design codes, such as Folded Reed-Solomon (FRS)

  (see also JLR'26)

- The Random Reed-Solomon (RRS) code

How good are the resulting FRS-based SNARKs?

… Currently, worse than provable RS-based SNARKs due to a large alphabet

… Can one improve the alphabet size of FRS ?? (maybe not [BCDZ'25, Thm 1.11])

Why Mutual Correlated Agreement? (MCA)

What is it good for?

An example problem on a linear code $\mathcal{C} \subseteq \mathbb{F}^n$

$\boldsymbol{P(s, t_0, t_1;\ u_0, u_1;\ c_0, c_1)}$ $\qquad$ $\boldsymbol{V^{u_0,u_1}(s, t_0, t_1)}$

$$\boldsymbol{s, u_0, u_1} \in \mathbb{F}^n, \quad t_0, t_1 \in \mathbb{F}, \quad \boldsymbol{c_0, c_1} \in \mathcal{C}$$

Goal: convince the verifier that

- $\text{dist}(\boldsymbol{u_i, c_i}) \leq \delta$, and
- $\langle \boldsymbol{c_i, s} \rangle = t_i$ for $i = 0,1$

prove $\boldsymbol{u_0, u_1}$ are δ-close to codewords that satisfy a linear constraint

Captures intermediate steps in Basefold and Whir

A simple IOP for this relation

$\boldsymbol{P}(\boldsymbol{s}, t_0, t_1;\ \boldsymbol{u_0}, \boldsymbol{u_1};\ \boldsymbol{c_0}, \boldsymbol{c_1})$

$\boldsymbol{V}^{\boldsymbol{u_0}, \boldsymbol{u_1}}(\boldsymbol{s}, t_0, t_1)$

$z \in \mathbb{F}$

$z \leftarrow \mathbb{F}$

Honest prover:

$\boldsymbol{c} := \boldsymbol{c_0} + z \cdot \boldsymbol{c_1}$

$\boldsymbol{c} \in \mathbb{F}^n$

$x_1, \ldots, x_\ell \leftarrow [n]$

Batch the two codewords into one

spot checks that $\boldsymbol{c}$ was computed honestly

$\boldsymbol{V}$ accepts if $\boldsymbol{c} \in \mathcal{C}$ and

- $\langle \boldsymbol{c}, \boldsymbol{s} \rangle = t_0 + z \cdot t_1$,
- $\forall i \in [\ell]$: $\boldsymbol{c}(x_i) = \boldsymbol{u_0}(x_i) + z \cdot \boldsymbol{u_1}(x_i)$

Complete when $u_i = c_i$, but is it knowledge sound?

An extractor to prove knowledge soundness

Because of spot checks, we know that w.h.p $|S| \geq (1-\delta)n$

$\Rightarrow$ because $\boldsymbol{c} \in \mathcal{C}$, we have $\boldsymbol{u_0} + z \cdot \boldsymbol{u_1}$ is (S, δ)-close to $\mathcal{C}$

$\Rightarrow$ by MCA, w.h.p both $\boldsymbol{u_0}, \boldsymbol{u_1}$ are (S, δ)-close to $\mathcal{C}$

So: $\mathbf{Ext}$ knows error locations in $\boldsymbol{u_0}, \boldsymbol{u_1}$ and can efficiently find $\boldsymbol{c_0}, \boldsymbol{c_1}$

$\mathbf{Ext}\big((\boldsymbol{u_0}, \boldsymbol{u_1}), (z, \boldsymbol{c})\big)$ ← takes V's input and an accepting transcript

- Let $S \subseteq [n]$ be the largest set such that $\Delta_S(\boldsymbol{c},\ \boldsymbol{u_0} + z \cdot \boldsymbol{u_1}) = 0$
- Erasure correct $\boldsymbol{u_0}, \boldsymbol{u_1}$ with respect to S to get codewords $\boldsymbol{c_0}, \boldsymbol{c_1}$
- Output $\boldsymbol{c_0}, \boldsymbol{c_1}$

Extraction theorem

Thm: $\Pr\left[\begin{array}{l}\text{Ext fails to output a}\\ \text{valid witness } \boldsymbol{c_0}, \boldsymbol{c_1}\end{array}\right] \leq \max\left(\varepsilon_{\text{mca}}(\mathcal{C}, \delta) + \frac{\Lambda(\mathcal{C}^{\equiv 2}, \delta)}{|\mathbb{F}|},\ (1-\delta)^t\right)$

See paper for proof

Need MCA and list decoding bound to hold for large δ

bigger δ ⇒ smaller t ⇒ shorter proof

Note: we need both MCA and list-decoding bounds for $\delta > J(\mathcal{C})$

$1,000,000

initiative by
ethereum foundation

in prizes to prove (or disprove!) Reed-Solomon proximity gaps conjectures … bound the MCA error above the Johnson bound

An initiative by the Ethereum Foundation to advance the foundations of modern zkVMs.

PRIZE JUDGES

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Help us solve it …

$1,000,000

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What we don’t want:
AI slop on proximity that confuses incoming researchers

Help us solve it …

To learn more:

Open Problems in List Decoding and Correlated Agreement

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Lists many additional interesting questions in this area, such as:

- **Can we build shorter SNARKs from Folded-RS (FRS) ?**
- **Non-RS codes: Better MCA bounds for LDPC codes?**

(random LDPC have MCA to capacity [GG'25])